THE RESISTANCE OF STRIP CONDUCTORS EMBEDDED IN A TWO-DIMENSIONAL FIELD DISTRIBUTION

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ABSTRACT

The a.c. resistance of strip conductors is investigated. For geometries involving a two-dimensional field distribution, one finds an interesting behavior: Between the DC and the skin-effect regime, there exists a third characteristic frequency range. The paper provides details on that range and points out consequences for circuit modelling. A simplified description is proposed.

STATEMENT OF PROBLEM

Nowadays, many high-frequency circuits are realized using miniaturized planar structures. Due to the small dimensions, conductor-loss effects gain importance and need to be included in circuit design. Typical examples are mm-wave monolithic integrated circuits as well as digital IC's with high packaging density. The conductors usually consist of metallic strips with rectangular cross-section. Since the strip dimensions range in the skin-depth's order of magnitude or below, loss analysis cannot be restricted to the conventional skin-effect case.

The classical theory assumes a one-dimensional structure, i.e., the parallel-plate geometry of Fig. 1). This setting enables one to derive an analytical description, the results of which are well known. Two characteristic

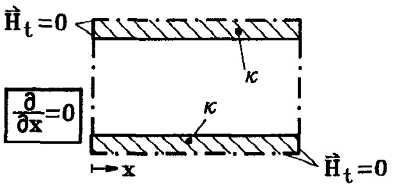

Fig. 1: Classical one-dimensional case: parallel-plate waveguide enclosed by magnetic walls (κ denotes strip conductivity).

frequency ranges can be distinguished: The skin-effect regime if skin depth δ remains small compared to strip thickness t and, for $\delta \gg t$, the DC limit. That principal finding has been utilized to develop loss descriptions with an extended range of validity, both in terms of frequency and line geometry. Examples are surface-impedance concepts (e.g. [1]) and the 'phenomenological loss equivalence method' (PLEM) [2].

The weak point of those treatments is that they basically assume the classical one-dimensional theory to be valid also in the two and three-dimensional case. Already in 1937, however, experimental results [3] for flat rectangular conductors indicated differences compared to the simple parallel-plate situation. This observation is the more important since today several widely used transmission-line structures possess inherently two-dimensional field distributions, e.g. the coplanar waveguide (CPW) and the coupled-strips configuration (CPS).

One way to overcome this uncertainty is to employ a more comprehensive approach (e.g. [4,5,6,7]). Such treatments, however, require great numerical efforts which render them unsuitable for many practical applications. Moreover, most of the publications cited deal with microstrip-like structures with a primarily one-dimensional nature.

The purpose of this paper is a threefold one:

- To demonstrate the influence of a 2D field distribution on strip resistance for common transmission-line geometries (CPW, CPS, and microstrip).
- To point out the consequences with regard to the modelling techniques in use.
- To provide an approximative description.

THE 2D CANONICAL CASE: CPS

Principal Results

In order to establish reliable reference data, we employ a rigorous full-wave analysis that holds within the entire range between DC and skin-effect frequencies [8]. Calculating the power dissipation in a conducting region and the corresponding current the resistance R per unit length can be obtained.

As a first example, the coupled-strip line according to Fig. 2 is studied. It combines predominant 2D field characteristics with a relatively simple geometry.

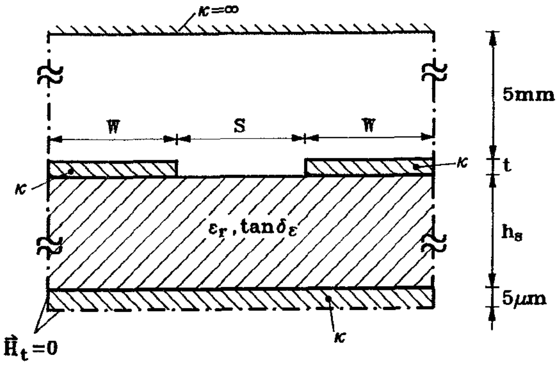

Fig. 2: Simple 2D case: coupled-strip line (CPS) with $W = 100\mu$m and $S = 20\mu$m; strip thickness $t = 1\mu$m, conductivity $\kappa = 3 \cdot 10^7$S/m, thickness $h_s = 600\mu$m (substrate properties ε_r and $\tan\delta_\varepsilon$ do not affect R under quasi-TEM conditions).

The frequency dependence of the resistance R is plotted in Fig. 3. One observes an interesting behavior:
As can be expected, for $t \geq 3\delta$ the classical skin-effect regime is detected with $R \sim \sqrt{f}$. For $f \leq 0.1$ GHz, on the other hand, R approaches the DC value. In between these two limits, however, a distinct third frequency range appears where neither the DC nor the simple skin-effect approximation holds ($f_l \leq f \leq f_{u2}$ in Fig. 3). In the following, it will be referred to as the *intermediate* range.

Thus, the second dimension indeed induces a new feature the classical one-dimensional theory fails to describe. One should note that the intermediate range covers the most part of the frequency band in Fig. 3 and hence it is of specific importance regarding microwave and mm-wave circuit elements.

When characterizing the intermediate range in more detail it is helpful to refer to the longitudinal current density J_z and its distribution over the strip cross-section: In the skin-effect limit, J_z flows primarily at the surface. The fields outside the metallizations closely resemble those for ideal conductors, i.e. they concentrate within the slot region.
As soon as skin depth $\delta = \sqrt{(2/\omega\mu\kappa)}$ reaches the same order of magnitude as strip thickness t, however, the field can penetrate the metallizations. Then, due to their 2D nature, the magnetic field and the current density J_z start to spread laterally, away from the slot region. Finally, for $f \to 0$, the uniform DC distribution is approached.

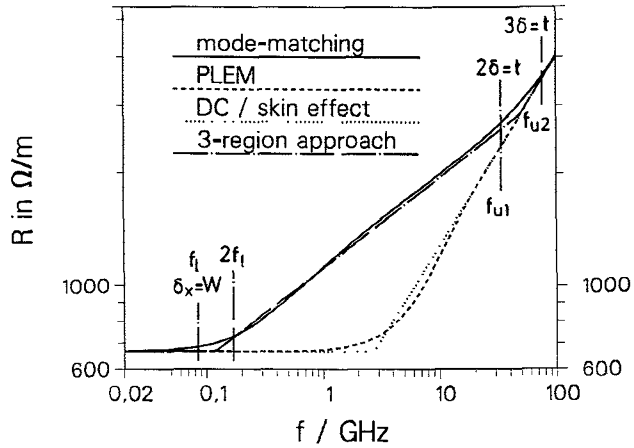

Fig. 3: CPS line resistance R as a function of frequency f (for line geometry see Fig. 2, bars refer to transition frequencies of eqn. 2):
- Mode-matching analysis [8] (solid)
- PLEM [2] (dashed)
- Simple DC - skin-effect curve (dotted)
- 3-region approach (eqn. 4, chain-dotted)

In a rather crude approximation, the lateral decay can be described by an effective depth δ_x (see [9]):

$$\delta_x = \frac{2}{\omega\mu\kappa t} \qquad (1)$$

Note that δ_x differs considerably from the classical skin depth $\delta = \sqrt{2/\omega\mu\kappa}$ and, additionally, depends on strip thickness t.

The main purpose of eqn. 1 is to determine the lower frequency limit f_l of the intermediate range, which occurs at $\delta_x = W$ with W being the metallization width. The upper limit f_u, on the other hand, denotes the lower bound of the classical skin-effect range. Hence f_u corresponds to the conditions $2\delta = t$ and $3\delta = t$, respectively (see also the bars in Fig. 3). Eqn. 2 summarizes the resulting relations between the transition frequencies and their physical meaning:

$$\begin{aligned} f_l &\Leftrightarrow \delta_x = W &&\Leftrightarrow f_l = \frac{1}{\pi\mu\kappa \cdot W \cdot t} \\ f_{u1} &\Leftrightarrow 2\delta = t &&\Leftrightarrow f_{u1} = \frac{4}{\pi\mu\kappa \cdot t^2} \\ f_{u2} &\Leftrightarrow 3\delta = t &&\Leftrightarrow f_{u1} = \frac{9}{\pi\mu\kappa \cdot t^2} \end{aligned} \qquad (2)$$

It follows that $f_{u1}/f_l \sim W/t$, i.e., the lower the strip aspect ratio t/W the larger the intermediate frequency range.

Modified Modelling Approach

Evaluating the results of Fig. 3 it is clear that the classical 1D theory has to be revised if 2D and 3D line geometries are to be investigated. Seeking for a simple description of R in the intermediate range between f_l and f_u, Fig. 3 indicates an interesting feature: The R curve follows approximately a straight line in the log-log scale, i.e.

$$R = c_1 \cdot f^{c_2} \tag{3}$$

with c_1 and c_2 being constants. Thus, given the DC and the skin-effect values of R together with the transition frequencies, a simple approximation for R can be derived. For the CPS structure, the best fit is found when choosing $\sqrt{2} f_l$ as the lower bound of the intermediate range and the geometric mean $\sqrt{f_{u2} \cdot f_{u1}}$ as the upper one.

$$R = \begin{cases} R_{DC} & \text{for } f \leq f_1 \\ R_{DC} \cdot (f/f_1)^{\nu} & \text{for } f_1 \leq f \leq f_2 \\ R_{SKE} \cdot \sqrt{f/f_2} & \text{for } f \geq f_2 \end{cases} \tag{4}$$

with R_{SKE} denoting the skin-effect value at $f = f_2$ and

$$\begin{aligned} f_1 &= \sqrt{2} f_l &&= \sqrt{2} \cdot \frac{1}{\pi \mu \kappa \cdot W \cdot t} \\ f_2 &= \sqrt{f_{u1} f_{u2}} &&= \frac{6}{\pi \mu \kappa \cdot t^2} \\ \nu &= \frac{\log\left(\frac{R_{SKE}}{R_{DC}}\right)}{\log\left(\frac{f_2}{f_1}\right)} \end{aligned}$$

The resulting curve is included in Fig. 3 by the chain-dotted line. It closely follows the mode-matching results. The curve behavior at the transition frequencies can be smoothened easily by adding terms quadratic in f that ensure continuity not only for R but also for $\partial R / \partial f$.

CPW RESULTS

As a further example to demonstrate the 2D phenomena, Fig. 4 presents typical data for a CPW structure. A 40μm wide and 1μm thick center strip is assumed as used in mm-wave MMIC's.

In analogy to the CPS, the frequency dependence of R can be described by a segmentation approach with 3 intervals according to eqn. 4. Only the transition frequencies f_1 and f_2 have to be modified. Optimum agreement is observed for

$$\begin{aligned} f_1 &= 2 f_l &&= \frac{4}{\pi \mu \kappa \cdot W \cdot t} \\ f_2 &= f_{u1} &&= \frac{4}{\pi \mu \kappa \cdot t^2} \end{aligned} \tag{5}$$

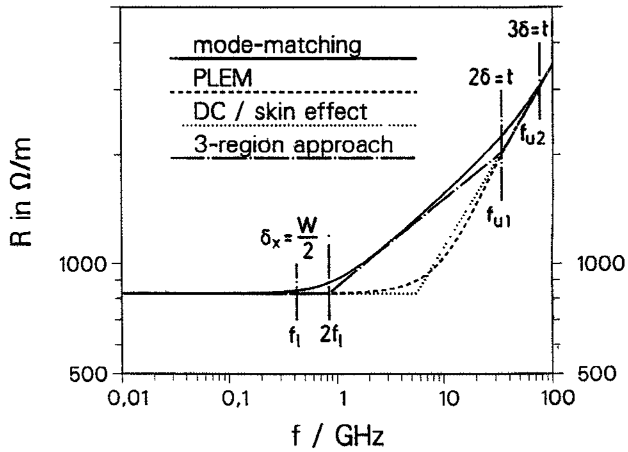

Fig. 4: CPW center conductor resistance R_c against frequency f: Comparison of mode-matching analysis [8] (solid), PLEM [2] (dashed), simple DC/skin-effect curve (dotted), and eqns. 4 and 5 (chain-dotted);
(line geometry equivalent to Fig. 2, center conductor width $W = 40\mu$m, slot width $S = 5\mu$m, ground-metallization width $W_g = 200\mu$m).

THE MICROSTRIP

Finally, the microstrip will be treated as an example for a line geometry with primarily 1D characteristics. Fig. 5 provides the cross-sectional data. The conductor dimensions equal those of the CPW center strip in Fig. 4. Substrate thickness h_s, however, is chosen very small (4μm), which results in a large ratio $W/h_s = 10$. For that case, the most part of the fields distributes rather homogeneously below the strip and closely resembles the 1D parallel-plate case of Fig. 1.

Consequently, as can be seen from Fig. 6, the influence of 2D effects on the overall performance is not as pronounced as in the CPW case. Nevertheless, one clearly observes deviations compared with the simple DC/skin-effect approximation. The discrepancies increase significantly at lower ratios W/h_s as Fig. 7 demonstrates for a structure with $W/h_s = 0.4$.

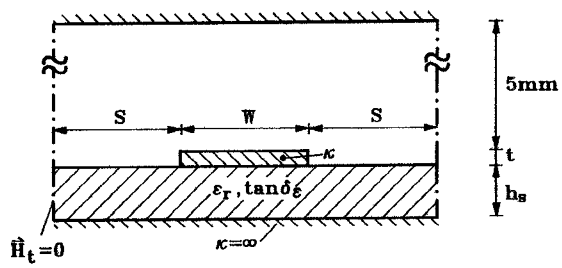

Fig. 5: The microstrip geometry used ($W = 40\mu$m, $S = 250\mu$m, $h_s = 4\mu$m, backside ideally conducting, material properties identical to Fig. 2).

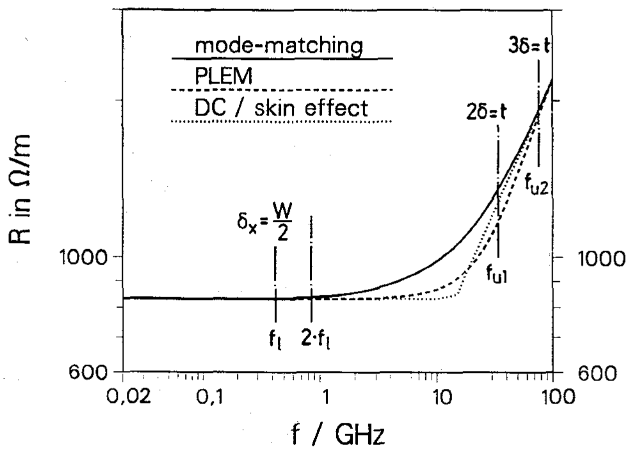

Fig. 6: Microstrip resistance R against frequency f (bars denote transition frequencies, for line geometry see Fig. 5):
- Mode-matching analysis [8] (solid)
- PLEM [2] (dashed)
- Simple DC - skin-effect curve (dotted)

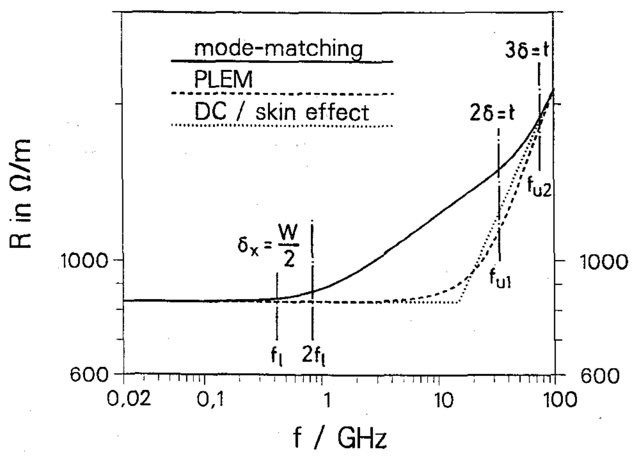

Fig. 7: Microstrip resistance R against frequency f for $W/h_s = 0.4$
(data of Fig. 6 except for $h_s = 100\mu$m).

CONCLUSIONS

The a.c. resistance of strip conductors embedded in a two-dimensional field distribution shows a principal difference compared to the classical one-dimensional theory: Between the DC and the skin-effect limit a distinct intermediate frequency range can be observed.

This finding applies particularly to miniaturized CPW and CPS structures as used in MMIC's. In that case, the intermediate range extends over a considerable part of the microwave and mm-wave spectrum.

Several loss-modelling techniques rely on one-dimensional theory, e.g. the surface impedance concepts that are in widespread use. Such approaches neglect the 2D effects described above and thus have to be checked carefully when employed in connection with lossy 2D line configurations.

Using the results of a comprehensive field-theoretical method as a reference, simplified descriptions for the intermediate frequency range can be developed that are particularly suited for practical circuit design.

REFERENCES

[1] T.E. van Deventer, P.B. Katehi, and A.C. Cangellaris, "High frequency conductor and dielectric losses in shielded microstrip," *1989 Int. Microwave Symposium Digest*, Vol. III, pp. 919-922.

[2] H.Y. Lee and T. Itoh, "Phenomenological loss equivalence method for planar quasi-TEM transmission line with a thin normal conductor or superconductor," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-37, pp. 1904-1909, Dec. 1989.

[3] S.J. Haefner, "Alternating-current resistance of rectangular conductors," *Proc. of the IRE*, Vol. 25, No. 4, pp. 434-447, April 1937.

[4] P. Silvester, "Modal network theory of skin effect in flat conductors," *Proc. of the IEEE*, Vol. 54, No. 9, pp. 1147-1151, Sept. 1966.

[5] P. Waldow and I. Wolff, "The skin-effect at high frequencies," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-33, pp. 1076-1082, Oct. 1985.

[6] G.I. Costache, "Finite-Element method applied to skin-effect problems in strip transmission lines," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-35, pp. 1009-1013, November 1987.

[7] R. Faraji-Dana and Y.L. Chow, "The current distribution and a.c. resistance of a microstrip structure," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-38, pp. 1268-1277, September 1990.

[8] W. Heinrich, "Full-wave analysis of conductor losses on MMIC transmission lines," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-38, pp. 1468-1472, October 1990.

[9] W. Heinrich, "Quasi-TEM description of MMIC coplanar lines including conductor-loss effects," *IEEE Trans. Microwave Theory Tech.*, to be published in January, 1993.